THERMAL POLARIZATION OF A VISCOUS GAS FLOW ACCOMPANIED BY SLIP IN A CHANNEL WITH A NONUNIFORM SURFACE

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A theoretical model of heat transfer is proposed and measurements are made of the temperature field in a gas flow in an isothermal channel with a nonuniform surface.

The author of [1] observed and studied the creation of a pressure gradient at the ends of a nonisothermal V-shaped tube filled with a gas. The gradient was proportional to the temperature difference between the ends of the tube and the site of the bend under the condition that the channels of the system had dissimilar surfaces. The tests were conducted on tubes of the same diameter made of different materials in the free-molecular regime of gas flow.

Analyzing this phenomenon from the viewpoint of the thermodynamics of irreversible processes, it is easy to conclude that there should exist an effect which is the counterpart to the Hobson effect. Meanwhile, both of these phenomena are probably characteristic of the entire range of Knudsen numbers. Thus, it should be expected that blowing an isothermal gas through the above-mentioned V-shaped tube will generate heat sources (sinks) at the bend as a result of the dependence of the mechanocaloric heat flow on the roughness of the surface. The phenomenon of the change in the temperature of a gas during its motion – often called thermal polarization – can be used in the diagnosis of gas flows.

Here, we theoretically describe the thermal polarization of an isothermal gas flow in a nonuniform channel in a regime with slip. The conclusions we make are checked experimentally.

We will examine the steady-state flow of a low-density (ideal) gas in a long isothermal tube at pressures corresponding to a viscous regime of motion with slip (Knudsen number Kn < 0.1). Heat and mass transfer in such a system can be described by equations of continuum mechanics and the equation of state of an ideal gas [2]:

$$\operatorname{div} \rho \mathbf{v} = 0, \tag{1}$$

$$\rho (\mathbf{v}\nabla)\mathbf{v} = -\operatorname{grad} P + \eta \Delta \mathbf{v} \to \left(\xi_0 + \frac{\eta}{3}\right) \operatorname{grad} \operatorname{div} \mathbf{v}, \tag{2}$$

$$\operatorname{div}\left(\rho \mathbf{v}\left(\frac{\mathbf{v}^2}{2} + \omega\right) - \mathbf{v}\overleftrightarrow{\sigma'} + \mathbf{q}\right) = 0, \tag{3}$$

$$P = \frac{\rho}{m} kT. \tag{4}$$

In traditional hydrodynamics [2], the heat flux $\mathbf{q}$ in Eq. (3) is assumed to be proportional only to ∇T. However, the kinetic theory of gases shows that it also contains a component connected with the longitudinal (along the tube axis) pressure gradient ∇P (the mechanocaloric heat flux). Thus, in accordance with [3], the heat flux should be written in the form

$$\mathbf{q} = -\varkappa \nabla T + L_{qv} \nabla P, \tag{5}$$

where $\nabla P = (0;\ 0;\ (\partial P/\partial z))$.

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In the general case, the coefficient of mechanocaloric heat flux L_{qv} is a function of the coordinates of the channel section and for small Knudsen numbers ($Kn \ll 1$) is determined through the coefficient of thermal creep A_T [3]:

$$L_{qv} = A_T \frac{\eta}{\rho}. \tag{6}$$

The tensor of the viscous stresses can be written in the form [2]

$$\sigma'_{ik} = \eta\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3}\delta_{ik}\frac{\partial v_l}{\partial x_l}\right) + \xi_0\delta_{ik}\frac{\partial v_l}{\partial x_l}. \tag{7}$$

We further assume that the channel has the form of a circular cylinder. In this case, the solution of the problem is independent of the azimuthal angle φ. By virtue of the impermeability of the channel walls and axial symmetry, the velocity components are equal: $v_\varphi = v_r = 0$; $v_z \equiv v$.

Using the following familiar relation for the thermal function of ideal gases [2]

$$\omega = C_p T, \tag{8}$$

we write Eqs. (1-4) in a cylindrical coordinate system:

$$\frac{\partial(\rho v)}{\partial z} = 0, \tag{9}$$

$$\frac{\partial P}{\partial r} = (\xi_0 + \eta/3)\frac{\partial^2 v}{\partial r \partial z}, \tag{10}$$

$$\rho v \frac{\partial v}{\partial z} = -\frac{\partial P}{\partial z} + \eta\Delta v + (\xi_0 + \eta/3)\frac{\partial^2 v}{\partial z^2}, \tag{11}$$

$$\rho v C_p \frac{\partial T}{\partial z} - \operatorname{div}\varkappa\nabla T + \frac{\partial}{\partial z}\left(L_{qv}\frac{\partial P}{\partial z}\right) = \frac{1}{2}\left[\left(\xi_0 + \frac{\eta}{3}\right)\frac{\partial^2 v^2}{\partial z^2} - \rho v\frac{\partial v^2}{\partial z} + \eta\Delta v^2\right], \tag{12}$$

$$P = \frac{\rho}{m} kT. \tag{13}$$

No analytical solution has been found for system (9-13) for arbitrary values of its parameters. However, for sufficiently small Reynolds numbers Re, corresponding to small pressure gradients in the channel, Eqs. (10) and (11) can be replaced by the equations of motion for an incompressible gas. Equation (12) is simplified if we assume that the channel has been thermostatted and that the temperature changes caused by the gas flow have the order $\Delta T \sim mv^2/k$ [2].

If we ignore terms of Eqs. (9)-(13) which are proportional to $Re(R/\bar{L})$; R^2/L^2, we can write the simpler system:

$$\frac{\partial(Pv)}{\partial z} = 0, \tag{14}$$

$$\frac{\partial P}{\partial r} = 0, \tag{15}$$

$$\frac{\partial P}{\partial z} = \eta\Delta_r v, \tag{16}$$

$$\Delta_r\left(\eta\frac{v^2}{2} + \varkappa T\right) = \frac{\partial}{\partial z}\left(L_{qv}\frac{\partial P}{\partial z}\right). \tag{17}$$

The typical boundary conditions of the problem are quite apparent:

$$\left.\frac{\partial v}{\partial r}\right|_{r=0} = 0; \quad \left.\frac{\partial T}{\partial r}\right|_{r=0} = 0, \tag{18}$$

$$v|_{r=R} = -\xi \left.\frac{\partial v}{\partial r}\right|_{r=R}, \tag{19}$$

$$-\varkappa \left.\frac{\partial T}{\partial r}\right|_{r=R} = \alpha (T - T_T)|_{r=R}, \tag{20}$$

$$P|_{z=0} = P_0; \quad P|_{z=L} = P_1. \tag{21}$$

In contrast to [2], we did not use the assumption of incompressibility in (17). Condition (19) allows us to consider the effect of the low density of the gas (viscous slip), while (20) presumes that heat transfer with the coefficient α takes place from the walls of the thermostat, which has the temperature T_T.

System (14)-(17) with boundary conditions (18)-(21) has the following solution:

$$v(r,\ z) = -\frac{(R^2 - r^2 + 2\xi R)}{4\eta}\frac{dP}{dz}, \tag{22}$$

$$P(z) = \sqrt{(P_0')^2 - [(P_0')^2 - (P_1')^2]\frac{z}{L}}\left(1 + \frac{4\xi}{R}\right), \tag{23}$$

$$T(r,\ z) = T_T - \frac{1}{\eta}\left(\frac{\partial P}{\partial z}\right)^2 \left\{\frac{(R^2 - r^2 + 4\xi R)(R^2 - r^2)}{32\varkappa} + \frac{\xi R^2}{4\alpha}\right\} + \frac{\partial}{\partial z}\left(L_{qv}\frac{\partial P}{\partial z}\right)\left(\frac{r^2 - R^2}{4\varkappa} - \frac{R}{2\alpha}\right), \tag{24}$$

where $P'(z) = P(z)\ (1 + 4\xi/R)$.

In the limit $Kn \to 0$ (24) changes into the expression

$$T(r,\ z) = T_T - \left(\frac{\partial P}{\partial z}\right)^2 \frac{(R^2 - r^2)^2}{32\eta\varkappa}, \tag{25}$$

which is quite different from the temperature field in a cylindrical channel for an incompressible fluid, presented in [2]:

$$T(r,\ z) = T_T + \left(\frac{\partial P}{\partial z}\right)^2 \frac{R^4 - r^4}{64\eta\varkappa}. \tag{26}$$

The reason for the change in the sign of thermal polarization in the transition to a compressible gas is apparently allowance for the cooling of the gas during its expansion along the flow.

It also follows from (25) that the heat flux on the wall iz zero:

$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = 0,$$

as a result of compensation for heating effects across the channel due to viscous friction and cooling during expansion. This is the reason for the independence of the temperature field (25) of the heat-transfer coefficient.

At finite Knudsen numbers, additional energy transport mechanisms are active and lead to redistribution of the temperature field. In particular, the heat flux to the wall is now nontrivial:

$$q|_{r=R} = -\varkappa\left.\frac{\partial T}{\partial r}\right|_{r=R} = \alpha (T - T_T)|_{r=R} = -\left(\frac{\partial P}{\partial z}\right)^2 \frac{\xi R^2}{4\eta} - \frac{\partial}{\partial z}\left(L_{qv}\frac{\partial P}{\partial z}\right)\frac{R}{2}. \tag{27}$$

It is evident from (27) that this heat flow is due to slip (the first term of (27)) and the mechanocaloric effect [the second term of (27)]. With a finite value of α, the given flow leads to a change in the temperature of the channel wall relative to the temperature of the thermostat. It is not hard to see that, in this case, viscous slip causes a reduction in wall temperature. The mechanocaloric effect can both increase and decrease wall temperature, depending on the form of the function $L_{qv} = L_{qv}(z)$.

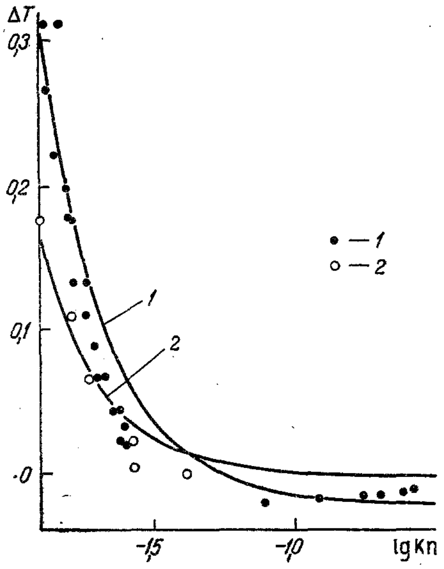

Fig. 1. Experimental and theoretical relations ΔT = f(log Kn) for helium (1) and xenon (2). log Kn is dimensionless; ΔT, K.

If L_{qv} changes linearly along the channel, then the maximum relative contribution of the third term of (24) is equal to about 10 Kn^2 and at Kn ≈ 0.1 will be about 10%. If the coefficient L_{qv} changes appreciably on a considerably shorter section of the channel (on the order of several channel diameters long), then the contribution of the term which accounts for the mechanocaloric effect may become decisive.

To check the validity of the proposed model of thermal polarization, we performed experimental measurements of the temperature of a gas in a channel with different pressure gradients.

The experiment was conducted on a glass capillary tube with a length L = 0.15 m and a radius $R = 1.1 \cdot 10^{-3}$ m. A local gradient L_{qv} was created inside the tube by abrading the inside surface on end sections with a length equal to L/3. Temperature was measured at the points of the largest gradient L_{qv}. Specifically, it was measured at distances L/3 and 2L/3 from the end of the channel. The measurement was made with wire copper-constantan thermocouples. We studied the dependence of the pressure difference at the points where the thermocouple junctions were located on the pressure P_0 at the tube inlet. The value of P_0 was varied within the range 30-2000 Pa. Experimental data for helium and xenon are shown in Fig. 1 in the form ΔT = f(log Kn). The figure also shows the theoretical relation ΔT = ΔT(log Kn) in accordance with Eq. (24) for the tube axis. We assumed that the coefficient A_T in Eq. (16) changed linearly over the length ΔL in the region of the points L/3 and 2L/3 where the rough section is replaced by the smooth section and vice versa. The value of A_T for different channel roughnesses and different gases was chosen in accordance with the data in [4]:

$$\mathrm{He}\, A_T^{\mathrm{sm}} = 1{,}0\, A_T^{\mathrm{r}} = 1{,}125,\quad \mathrm{Xe}\, A_T^{\mathrm{sm}} = A_T^{\mathrm{r}} = 1{,}125. \tag{28}$$

In constructing the theoretical dependence, we made a correction for the heating of the thermocouples in a free-molecular gas flow [5]:

$$\Delta T' = \frac{mv^2}{k}\,\frac{\alpha_t(2-\alpha_t)}{\alpha_n + \alpha_t(2-\alpha_t)}, \tag{29}$$

where the quantity mv^2/k is the temperature gradient for the diffuse core of junction surface scattering, while the multiplier $B = \alpha_t(2 - \alpha_t)/[\alpha_n + \alpha_t(2 - \alpha_t)]$ corresponds to the fraction of atomic accomodation energy due to accomodation of tangential momentum on the surface of the thermocouple.

The satisfactory agreement between the experimental data and results calculated from Eq. (24), evident in Fig. 1, was obtained with physically realistic values of the parameters βL = 2.4 mm; $B_{He} = 0.36$; $B_{Xe} = 0.41$.

An interesting feature of the dependence of thermal polarization on the Knudsen number is the change in the sign of the temperature difference. This is particularly clear in the case of helium. The proposed model (24) links this change with the possibility that, at Kn ~ 0.1, the mechanocaloric heat flow becomes decisive.

The temperature of the first thermocouple junction (in the direction of the flow) is higher, since the total cooling of the gas in the core of the flow is proportional to the square of velocity and the latter increases with a decrease in the density of the gas. With a reduction in pressure at the inlet, the contribution of the mechanocaloric heat flow increases. This contribution may become decisive in a channel with a rough surface for gases characterized by a substantial difference between the coefficients of thermal creep A_T for smooth (A_T^{sm}) and rough (A_T^r) surfaces (helium, neon). Also, since the direction of the mechanocaloric heat flow is opposite the direction of the gas flow and $A_T^{sm} < A_T^r$, then the first junction will be cooled and the second heated — leading to the observed change in the sign of the temperature difference.

It should be noted that by recording the temperature field of the flow and the temperature of the channel walls, it is possible to use (24) to evaluate flow velocity, the flow regime, and certain other individual characteristics of gases moving in these channels.

NOTATION

ρ, density of the gas; v, gas velocity vector; P, gas pressure; η, ξ_0, shear and bulk viscosity of the gas, respectively; ω, enthalpy of a unit mass; $\overleftrightarrow{\sigma}'$, tensor of the viscous stresses; q, heat flux; m, mass of a gas molecule; κ, thermal conductivity of the gas; L_{qv}, coefficient of mechanocaloric heat flux; C_p, isobaric heat capacity of the gas; α, heat-transfer coefficient; $\xi = \sigma\lambda$, σ, coefficient of viscous slip; λ, mean free path in the gas; L, R, length and radius of capillary tube; A_T^{sm}, A_T^r, coefficient of thermal creep for smooth and rough surfaces, respectively; Kn, Re, Knudsen and Reynolds numbers; T, T_T, temperatures of the gas and the thermostat, respectively; r, φ, z, cylindrical coordinates; α_n, α_t, coefficients of energy accomodation and tangential momentum, respectively; k, Boltzmann constant; $\Delta_r = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})$.

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